**Make Sense of Problems Persevere in Solving Them**

 *Mathematically proficient students:*

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| **Begin by…**   * **…explaining the problem to themselves, looking for entry points.** * **…analyzing what is given/not given, analyzing relationships, determining goals of the problem.** * **…making conjectures about solutions, planning solution pathways.** * **…considering similar/simpler problems, trying special cases.** | **Solve by…**   * **…monitoring and evaluating progress, changing course if necessary.** * **…creating, then comparing and explaining connections between equations, tables, graphs.** * **…drawing diagrams of important relationships.** * **…searching for regularity or trends.** | **Finalize by…**   * **…checking answers to problems using different methods.** * **…continually asking, “Does this make sense?”** * **…understanding approaches of others to solving complex problems** * **…identifying connections and comparing different approaches.** |

**Reason Abstractly and Quantitatively**

 *Mathematically proficient students:*

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| * **MAKE SENSE of Quantities (amounts, numbers, sizes) and their relationships in problem situations by…** | * **REASON Quantitatively (with amounts, numbers, sizes) to create a clear representation of the problem by…** |
| **Contextualizing:**   * **…giving the numbers a context (background, labels, situations, locations).** * **…pausing during numeric or symbolic manipulation to probe into the background or situation.**   **De-contextualizing:**   * **…representing a problem using abstract numbers and symbols.** * **…manipulating symbols outside a problem context.** | * **…considering the units (labels) for the numbers (i.e., yards or miles).** * **…attending to the meaning of quantities, not just manipulating the numbers or symbols.** * **…flexibly using different properties of operations and objects.** |

**Construct Viable Arguments Critique Others’ Reasoning**

*Mathematically proficient students:*



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| **Construct:**   * **Construct arguments using previously established assumptions, definitions and results.** * **Make conjectures. Build a logical progression of statements to explore the truth of conjectures.** * **Analyze situations by breaking them into cases. Recognize and use counterexamples.** * **Justify conclusions. Communicate conclusions to others.** * **Reason inductively (generalize from individual cases). Make plausible arguments involving contexts.** | **Critique:**   * **Listen to or read arguments of others, decide whether they make sense, ask questions to clarify or improve the arguments.** * **Compare the effectiveness of two arguments.** * **Distinguish correct logic or reasoning from that which is flawed.** * **Explain a flaw in an argument.** |

**Model with Mathematics**

*Mathematically proficient students:*



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| **Use models to *solve problems* in everyday life, society, and work.**  **For Example:**   * **In sixth grade, model a proportion or percentage problem by drawing bars.** * **In middle grades, apply proportional reasoning models to plan a school event or analyze a community problem.** * **In high school, use geometric models to solve a design problem or use a function to show a real-world relationship.** * **In real-life, model a scheduling problem using a network.** * **Identify, map and interpret relationships in real-world situations using diagrams, two-way tables, graphs, flowcharts, formulas, etc.** | **Use models to *understand and communicate* mathematical ideas.**   * **Create a model to show the meaning of a percentage problem or an equation.** * **Move interchangeably among models: that is concrete manipulatives, drawings, equations, tables, graphs, flowcharts, formulas.** * **Abstract the mathematics from the models or create the models to explain the mathematics.** * **Use models to simplify a situation and to make assumptions and approximations.** * **Analyze and draw conclusions about mathematical relationships using diagrams, tables, graphs, flowcharts.** * **Reflect on results and improve the model if needed.** |

**Use Appropriate Tools Strategically**

*Mathematically proficient students:*



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| **At all age levels:**   * **Consider available tools when solving a mathematical problem (pencil, paper, concrete model, ruler, protractor, calculator, spreadsheet, computer software).** * **Make sound decisions about when tools might be helpful.** * **Recognize insight to be gained and limitations of tools. For example, students detect possible errors (from graphing calculators) using estimation and other mathematical knowledge.** | **C:\Documents and Settings\CurricStaff\Local Settings\Temporary Internet Files\Content.IE5\C5QF0TEJ\MC900229685[1].wmf** | **At appropriate age levels:**   * **Use technology to visualize results of assumptions, explore consequences, and compare predictions.** * **Identify relevant mathematical resources, such as digital content on a website, and use them to pose or solve problems.** * **Use technological tools to explore and deepen their understanding of concepts.** |

**Attend to Precision**

*Mathematically proficient students:*



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| **Communicate by…**   * **…speaking, reading, and writing precisely to others.** * **…using clear definitions in discussion with others and in their own reasoning.** * **…stating the meaning of the symbols used, including using the equal sign consistently and appropriately.** * **…specifying units of measure carefully.** * **…labeling axes with units of measurement to clarify relationships of quantities in a problem.** | **Calculate and Problem Solve by…**   * **…calculating accurately and efficiently.** * **…expressing numerical answers with precision appropriate for the problem context.** * **…making certain solution matches the problem asked.** * **…estimating to check if computed solution is reasonable and therefore precise.** |

**Look for and Make**

**Use of Structure**

*Mathematically proficient students:*



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| * **Look closely to observe patterns or structure, for example:** * ***Young students* might see that three + seven more is the same amount as seven + three more, or sort shapes according to number of sides.** * ***Later*, students will see 7 x 8 equals (7 x 5) + (7 x 3), in preparation for the Distributive Property.**      * **In the expression *x*2 + 9*x* + 14, *older students* can see the 14 as (2 x 7) and the 9 as (2 + 7).** * **Step back for a big picture look and shift perspective.** * **See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.** |  |

**Look for, Express Regularity**

**in Repeated Reasoning**

*Mathematically proficient students:*



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| * **Notice if calculations are repeated and look both for general methods and for shortcuts, for example:** * ***Upper elementary students* might notice when dividing 25 by 11 that they are repeating the same calculations over and over and recognize a repeating decimal.** * ***Middle school students* might use a graphing calculator to observe a series of lines in slope/intercept form (*y*=m*x*+b), paying attention to what happens when “m” or “b” change, then drawing conclusions.** * ***High school students* might notice the regularity in the way terms cancel when expanding (*x* – 1)(*x* + 1), (*x* – 1)(*x*2 + *x* + 1), and (*x* – 1)(*x*3 + *x*2 + *x* + 1) might lead students to the general formula for the sum of a geometric series.** | * **Work to solve a problem—keep the problem in mind while working on details.** * **Continually evaluate reasonableness of intermediate results during the problem solving process.** |